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# UNDERSTANDING CONSUMER BEHAVIOUR AND CONSUMPTION

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*Abstract:* **This paper is a review of consumer behaviour with respect to linear expenditure system, separable and additive utility functions, homogeneous and homothetic utility functions, direct and indirect utility functions and duality in consumption. Consumers of economic commodities are believed to be rational (always seeking to maximize utility and minimize cost). This theory of consumer behavior is useful in the analysis of utility optimization. The specific properties of utility functions are of interest to the economist. Linear expenditure systems are helpful in developing theories that are conformable to empirical modeling and estimation. Separable and additive utility functions emphasize that utility is derived from a group of commodities. Homothetic function is a monotonic transformation of a homogeneous function. The concept of' 'Duality' enables analysis of direct and indirect utility functions based on convenience. We are able to derive the compensated demand function using the duality principle.**

*Keywords:* **Consumer behavior, linear expenditure, utility function, direct, indirect, duality, homogeneous, homothetic, separability, additivity**

## 1. INTRODUCTION

The theory of consumer behavior touches on the attitude of consumers to economic goods. The theory of consumer's behavior finds applications in the consumer's effort to attain optimal utility at varying levels of income and prices. This behaviour of the consumer is better explained using his demand (consumption of) for basket of goods determined by the level of satisfaction (utility) he derives. By extension therefore, consumer behavior is easily explained using utility function. These utility functions are assumed to have peculiar features such as quasi-concavity, differentiability and so on. It becomes necessary to study the implications of these assumptions on the consumer's utility function in a more detailed manner. This paper is a review of selected areas under the theory of consumer behavior namely linear expenditure system, separable and additive utility functions, homogeneous and homothetic utility functions and lastly indirect utility functions and duality in consumption.

## 2. LINEAR EXPENDITURE SYSTEMS

Linear expenditure systems (LES) are models that deal with baskets (groups) of commodities rather individual commodities. Addition of these groups result in total expenditure of consumer. They are very useful in aggregate econometric modeling as they make it possible for the consumption function to be disaggregate as may be desired. Usually, the linear expenditure systems (LES) are developed on the basis of utility function from which demand function is normally derived by maximizing the utility function when there is budget constraint. By this approach, LES is similar to the models based on indifference curves.

Linear expenditure systems are helpful in developing theories that are amenable to empirical modeling, and hence, estimation. This is illustrated in the following example with the Klein-Rubin (Stone-Geary) utility function

$$U = \alpha_1 \text{In} (q_1 - \gamma_1) + \alpha_2 \text{In} (q_2 - \gamma_2) \qquad (1)$$

Such that $q_1 > \gamma_1$ and $q_2 > \gamma_2$ . The $\gamma$'s are positive and could described as minimum subsistence quantities. The α's are equally positive; by positive monotonic transformation, U' = U/( $\alpha_1 + \alpha_2$), we obtain

$$U' = \beta_1 \text{ In } (q_1 - \gamma_1) + \beta_2 \text{ In } (q_2 - \gamma_2) \qquad (2)$$

The coefficients $\beta_1$ and $\beta_2$ ($\beta_1 + \beta_2 = 1$) are called "share" parameters.

Introducing the constraint, we now have the function

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$$

$$Z = \beta_1 \text{ In } (q_1 - \gamma_1) + \beta_2 \text{ In } (q_2 - \gamma_2) + \lambda(y - p_1 q_1 - p_2 q_2) \qquad (3)$$

By setting the first partial derivatives equal to zero

$$\frac{\partial Z}{\partial q_1} = \frac{\beta_1}{q_1 - \gamma_1} - \lambda p_1 = 0$$

$$\frac{\partial Z}{\partial q_2} = \frac{\beta_1}{q_2 - \gamma_2} - \lambda p_2 = 0$$

(4a – c)

$$\frac{\partial Z}{\partial \lambda} = y - p_1 q_1 - p_2 q_2 = 0$$

By solving equations (4a – c) for optimality, we obtain the demand functions

$$q_1 = \gamma_1 + \frac{\beta_1}{p_1} (y - p_1 \gamma_1 - p_2 \gamma_2) = 0$$

(5a - b)

$$q_2 = \gamma_2 + \frac{\beta_2}{p_2} (y - p_1 \gamma_1 - p_2 \gamma_2) = 0$$

We can obtain the expenditure function by multiplying equation (5a) and (5b) by $p_1$ and $p_2$ respectively

$$p_1 q_1 = p_1 \gamma_1 + \beta_1 (y - p_1 \gamma_1 - p_2 \gamma_2)$$

(6a – b)

$$p_2 q_2 = p_2 \gamma_2 + \beta_2 (y - p_1 \gamma_1 - p_2 \gamma_2)$$

Equations (5a) and (5b) are linear in both income and price, and they are amenable to linear regression analysis. However, LES differs from the models based on indifference curves by the fact that it (LES) applies to 'groups of commodities' among which there is no possibility of substitutes while the indifference curves approach analyzes commodities that are substitutes.

## 3. SEPARABLE AND ADDITIVE UTILITY FUNCTIONS

Utility functions are assumed to be regular, quasi-concave, differentiable and increasing. Separable utility function is an aspect of linear expenditure systems. It holds that utility is derived from a group of commodities. Separable utility function is one in which the quantity consumed of argument (variable or commodity) does not affect the result of changing another. For example, given that utility, *U*, is a function of consumption of two products, *m* and *n*, in a separable utility function, this can be divided into two separate parts as follows:

$$U_{(m, n)} = f(m) + g(n) \qquad (7)$$

Therefore, the marginal utility of *m* is independent of *n*, and the marginal utility of *n* is independent of *m*. That is, $U_{mn} = 0$. This also holds if utility is an increasing function of consumption, *c*, and a decreasing function of work done, *z*. Assuming such function is separable, it can be written as

$$U_{(c,\,z)} = f(c) - g(z) \qquad (8)$$

so that $U_{cz} = 0$.

In summary, a utility function is strongly separable in all its variables if it can be written as

$$U = F[\boldsymbol{\Sigma} f_i(q_i)\,] \qquad (9)$$

Where $F$ and $f_i$ are increasing functions. An example is given by the function

$$U = \text{In}\,(q_1{}^{\alpha} + q_2{}^{\beta} + q_3{}^{\gamma}\,) \qquad (10)$$

Separable functions are very convenient for fitting mathematical models but very difficult and complex as models of human behavior.

Since substitutability of groups of commodities is not possible with the LES, then the utility function is additive (i.e. total utility 'U' equals sum of the individual utilities from the various groups of commodities), and the indifference map would appear as in figure 1.

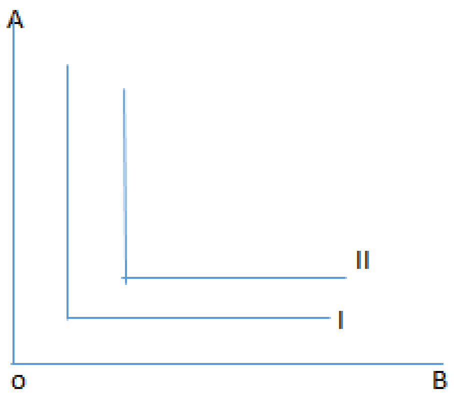


**Fig. 1: Indifference map for complementary goods**

If for example, we assume that all the commodities bought by consumers are grouped into five:

A Clothing

B Kitchen ware

C Food and beverages

D Household operation and maintenance expenses

E Transport, power and communication

The total utility 'U' is given by

$$U = \Sigma\, U_i \qquad (11)$$

$$U = U_{(A)} + U_{(B)} + U_{(C)} + U_{(D)} + U_{(E)} \qquad (12)$$

A utility function is strongly additive if it can be written as

$$U = \Sigma\, f_i(q_i) \qquad (13)$$

where the $f_i$ are increasing. Additivity is a special case of separability. Any utility function that has a monotonic transformation which is additive could be treated as being additive for all theorems applicable to additive functions. An illustrative example is given by the function

$$U = q_1{}^{\alpha} + q_2{}^{\beta} + q_3{}^{\gamma} \qquad (14)$$

The function $U = q_i^{\alpha} q_2$ is separable but does not appear to be additive. However, when transformed into natural logarithm as $F(U) = \alpha$ In $q_1$ + In $q_2$, it becomes additive. This also applies to the antilog of $U$ = In ($q_1{}^{\alpha} + q_2{}^{\beta} + q_3{}^{\gamma}$) which is strongly additive. The assumption of strong separability allows a pair-wise analysis that are not easily possible in the general case.

Another feature of an additive utility function is that all cross partials equal to zero. That is,

$$\frac{\partial U}{\partial q_i \partial q_j} = 0 \qquad\qquad for\ all\ i\ \neq j,$$

with the regular strict quasi-concavity condition as $f_{11}f_2{}^2 + f_{22}f_1{}^2 < 0$ where two variables are involved.

A utility function is said to be weakly separable if the variables can be partitioned into two or more groups ($q_1$, ……., $q_k$) + ($q_{k+1}$, ……… , $q_n$) in such a manner that

$$U = F[f_1(q_1, \ldots\ldots, q_k) + f_2(q_{k+1}, \ldots\ldots, q_n)] \qquad (15)$$

and weakly additive if

$$U = f_1(q_1, \ldots\ldots, q_k) + f_2(q_{k+1}, \ldots\ldots, q_n) \qquad (16)$$

Separability in here means that the Rate of Commodity Substitution (RCS) for a pars of variables within the same group are not affected by quantities for variables outside the group. Additivity implies that cross partials for pairs of commodities in different groups are identically zero.

## 4. HOMOGENEOUS AND HOMOTHETIC UTILITY FUNCTIONS

A function is said to be homogeneous if multiplying all the arguments by any constant λ, multiplies the value of the function by $\lambda^k$. Thus function is homogeneous of order $k$. A utility function is homogeneous of degree $k$ if

$$f(tq_1, \ldots\ldots, tq_n) = t^k f(q_1, \ldots\ldots, q_n) \qquad (17)$$

where $k$ is a constant and $t$ is any positive real number such that ($tq_1$, ……. , $tq_n$) is within the domain if the function. If a function is homogeneous of degree $k$, its partial derivatives are homogeneous of degree $k - 1$. This illustrated by differentiating equation (17), applying the function of a function rule:

$$f(tq_1, \ldots\ldots, tq_n) = t^k f(q_1, \ldots\ldots, q_n)$$

Therefore, the rate of commodity substitution (RCS) for $q_1$ and $q_2$ is obtained as:

$$\frac{tf_i(tq_1, \ldots . tq_n)}{tf_j(tq_1, \ldots . tq_n)} = \frac{t^{k-1}f_i(tq_1, \ldots . tq_n)}{t^{k-1}f_j(tq_1, \ldots . tq_n)} = \frac{f_i(tq_1, \ldots . tq_n)}{f_j(tq_1, \ldots . tq_n)}$$

The RCS does not vary with proportionate changes in consumption levels, implying that a consumer that is indifferent between two consumption baskets, will also be indifferent between any other two baskets that use the multiple of the first pair.

Indifference curve corresponding to two different utility functions are identical if one of the functions is a monotonic increasing function of the other.

Homothetic Utility Function: A function is called a homothetic function, if it is a monotonic transformation of a homogeneous function. Features of homogeneous functions are exhibited by all functions that are monotonic increasing functions of homogeneous functions. Therefore, utility functions that fall within this class (including homogeneous functions), are said to be homothetic. For homothetic utility functions, rate of commodity substitution will depend of relative rather than absolute commodity quantities. Therefore, a careful consideration of the equation of rate of commodity substitution (RCS) can give an indication as to whether a specific function is homothetic or not. For instance, $U$ = A function is called a homothetic function, if it is a monotonic transformation of a homogeneous function.

## 5. INDIRECT UTILITY FUNCTIONS AND DUALITY IN CONSUMPTION

The dual approach to demand theory is derives from the fact that preferences can be represented in two forms other than the utility function; these include the expenditure function and the indirect utility function. Let U: $R_+^I$ …….. $R_+$ be an upper semi-continuous unbounded utility function. The expenditure function e: $R^I_{++}$ x $R_+$ ......... $R_+$ generated by $U$ is defined by $e(p,u)$ = min $p$ . $x$ subject to $U(x) \geq u$. The indirect utility function $V$: $R^{I}_{++}$ x $R_{+}$ ......... $R_{+}$ generated by U is defined by $V(\text{p}, I)$ = max $U(x)$ subject to p . x $\leq I$

The usefulness of the dual approach results from two facts: First, the Marshallian demand function can be computed from the indirect utility function by differentiation. Second. The Hicksian demand function can be computed from the expenditure function by differentiation.

Indirect utility function gives the maximum utility as a function of normalized prices. The direct utility function gives a description of the preferences regardless of market forces. The indirect utility function is a reflection, to some extent, of optimization and market prices

For any utility function $U$(x), the corresponding indirect utility function is given by:

$V(p,w) \equiv \max \; \{\underline{U}(x) | x \geq 0, px \leq w\}$

$\equiv \max \; \{U(x) | x \in B_{p,w}\}$,

so that if x∗ is the solution to the UMP, then V(p,w)=U(x∗).

Since V (p,w) ≡ max $\{U(x) | x \geq 0, px \leq w\}$

and x(p,w) ≡ argmax $\{U(x) | x \geq 0, px \leq w\}$,

so that

$V(p,w) \equiv U(x(p,w))$.

If for example, we are given that $v_i = p_i/y$. then we can express the budget constraint as

$$1 = \sum_{i=1}^{n} v_i q_i \tag{18}$$

Usually, optimal solutions are known to be homogeneous and of degree zero in income and prices, therefore, nothing has changed by the transformation of budget equation to 'normalized' prices. We will obtain the first order condition for utility maximization by combining the utility function $U = f(q_1, \ldots, q_n)$ with equation (18):

$$f_i - \lambda v_i = 0 \qquad (\text{for } I = 1, \ldots\ldots, n) \tag{19}$$

$$1 - \sum_{i=1}^{n} v_i q_i = 0$$

By solving equation (19), we obtain the ordinary demand function:

$$q_i = \boldsymbol{D}(v_1, \ldots, v_n) \tag{20}$$

The indirect utility function $g(v_1 \ldots v_n)$ can be expressed as

$$U = f[D_1(v_1, \ldots, v_n), \ldots\ldots, D_n(v_1, \ldots v_n)] = g(v_1, \ldots\ldots v_n) \tag{21}$$

By applying the composite function rule to eq (20),

$$g_i = \sum_{i=1}^{n} f_i \frac{\partial q_i}{\partial v_j} = \lambda \sum_{i=1}^{n} v_i \frac{\partial q_i}{\partial v_j} \qquad j = 1, \ldots \ldots n \tag{22}$$

A partial differentiation of eq (18) with respect to $v_1$ gives the *Roy's Identity*

$$\sum_{i=1}^{n} v_i \frac{\partial q_i}{\partial v_j} = -q_i \qquad j = 1, \ldots \ldots n$$

From eq (22),

$$q_i = -\frac{g_j}{\lambda} \qquad j = 1, \ldots \ldots . . n \tag{23}$$

Equation (23) is the *Roy's Identity.* Optimal demand relates to the derivatives of the indirect utility function and optimal value of the Langrange multiplier, λ (marginal utility of income).

Considering an optimization problem such that equation (20) is minimized, subject to eq (18) with normalized prices as variables and quantities as parameters, we form the function,

$$Z = g(v_1,\ldots\ldots v_n) + \mu(\sum_{i=1}^{n} v_i q_i - 1)$$

Setting the partial derivatives equal to zero:

$$\frac{\partial Z}{\partial v_i} = g_i - \mu q_i = 0 \qquad\qquad i = 1,\ldots., n$$

$$\frac{\partial Z}{\partial \mu} = \sum_{i=1}^{n} v_i q_i - 1 = 0 \qquad (23)$$

By solving equation (23) expressing prices as functions of quantities, we obtain the *Inverse Demand Functions.*

$$v_i = V_i(q_i, \ldots\ldots, q_n) \qquad (24)$$

## 6. DUALITY THEOREMS

A set of duality theorems describe the relationship between direct and indirect utility functions.

**Theorem 1** Let $f$ be a finite regular strictly quasi-concave increasing function which obeys the interior assumption. The indirect utility function, $g$, determined from $f$ will be a finite regular strictly quasi-convex decreasing function for positive price.

**Theorem 2** Let $g$ be a finite regular quasi-convex decreasing function in positive prices. The direct utility function, $h$, determined from the $g$ will be a finite regular strictly quasi-concave increasing function.

**Theorem 3** Following the assumptions above,

$$h(q_1, \ldots\ldots, q_n) = g\left[V_{i(}q_1, \ldots\ldots, q_n\right), \ldots\ldots, V_{n(}q_1, \ldots\ldots, q_n)]$$

and

$$g(v_1, \ldots\ldots, v_n) = h\left[D_{i(}v_1, \ldots\ldots, v_n\right), \ldots\ldots, D_{n(}v_1, \ldots\ldots, v_n)]$$

Duality in consumption provides a closer link between demand and utility functions for empirical studies. It is possible to go from demand functions to the indirect utility function, using Roy's Identity, and then to the corresponding direct utility function. Duality finds application in comparative statics analyses. Through duality theory, we are able to obtain the counterparts for homotheticity, separability and additivity. We are therefore, able to conduct many theoretical analyses using either the direct or indirect utility functions, depending on convenience.

## 7. UTILITY-EXPENDITURE DUALITY

Looking at the minimization of expenditures required to attain a level of utility, the values obtained for the utility maximizing quantities ($q_i$) represents the *compensated demand functions.* By plucking the values of $q_i$ in $\sum_{i=1}^{n} v_i q_i$ , we get the minimum expenditure function, $E(p_1,\ldots., p_n,U^0)$ required to achieved a given level of utility. It is verifiable that the expenditure function obtained above is homogeneous of degree one in prices and monotone increasing in $U^0$. According to Shepherd's Lemma, the partial derivative of the expenditure function with respect to the $i$th price is the $i$th compensated demand function.

Supposing our demand function is given by $q_i = q_i(p_1, \ldots\ldots, p_n, U°)$ then

$$E(p_1, \ldots\ldots, p_n, U) = \sum_{i=1}^{n} p_i q_i\,(p_1, \ldots\ldots, p_n, U°)$$

Taking a partial derivative of $E$,

$$\frac{\partial E q_i}{\partial p_i} = q_i(p_1, \ldots\ldots, p_n, U°) + \sum_{j=1}^{n} p_1 \frac{\partial q_j(p_1, \ldots., p_n, U°)}{\partial p_i}$$

However, compensated demand is obtained by expenditure minimization for a given level of utility $U°$, therefore, change in total expenditure arising from a small change in price will be zero. This implies that the second term $(\sum_{j=1}^{n} p_1 \frac{\partial q_j(p_1,....,p_n,U°)}{\partial p_i})$ in the equation above equals zero, and can now be dropped. Thus,

$$\frac{\partial E q_i}{\partial p_i} = q_i(p_1, ......, p_n, U°).$$

It is good to point out that the duality relationship between utility and expenditure functions is similar to the duality between production and cost functions.

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